CP violation in scatterings, three body processes and the Boltzmann equations for leptogenesis

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Abstract: We obtain the Boltzmann equations for leptogenesis including decay and scattering processes with two and three body initial or final states. We present an explicit computation of the CP violating scattering asymmetries. We analyze their possible impact in leptogenesis, and we discuss the validity of their approximate expressions in terms of the decay asymmetry. In scenarios in which the initial heavy neutrino density vanishes, the inclusion of CP asymmetries in scatterings can enforce a cancellation between the lepton asymmetry generated at early times and the asymmetry produced at later times. We argue that a sizeable amount of washout is crucial for spoiling this cancellation, and we show that in the regimes in which the washouts are particularly weak, the inclusion of CP violation in scatterings yields a reduction in the final value of the lepton asymmetry. In the strong washout regimes the inclusion of CP violation in scatterings still leads to a significant enhancement of the lepton asymmetry at high temperatures; however, due to the independence from the early conditions that is characteristic of these regimes, the final value of the lepton asymmetry remains approximately unchanged.

Keywords: Baryogenesis, CP violation, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Leptogenesis [1] provides a very attractive scenario to generate the baryon asymmetry of the Universe. Indeed, the existence of heavy right handed neutrinos is strongly motivated by the see-saw mechanism [2] introduced to generate the observed light neutrino masses. The Majorana nature of the right handed neutrinos implies violation of lepton number, the large values of their masses imply that the out of equilibrium requirement can hold when they decay, and moreover these decays can be CP violating due to the phases that are generally present in the Yukawa couplings with the lepton doublets. Since the three Sakharov conditions [3] can be met, the question then is a quantitative one, i.e. whether the asymmetry generated in these scenarios is large enough to account for the observed value.

In recent years, quantitative analyses of leptogenesis have become more and more sophisticated, taking into account many subtle but significant ingredients, such as several washout processes [4, 5, 7, 8], proper subtraction of on-shell heavy neutrino intermediate states [9], thermal corrections to particle masses, couplings and decay asymmetries [9], spectator processes [10, 11], flavor effects [12–19], and the possible effects of the heaviest right handed Majorana neutrinos $N_{2,3}$ [12, 20–22] (for reviews of the most recent results see [23]).

The aim of this work is to discuss in some detail the inclusion of CP violation in scatterings, and of processes involving two and three body initial or final states. In section 2 we derive the Boltzmann Equations (BE) involving these terms and obtain a general parameterization for them. In section 3 we present a detailed computation of the CP violating asymmetries in scatterings in zero temperature field theory. These asymmetries

have been considered before in the limit in which they are proportional to the CP asymmetry in decays [7, 16], and we discuss here the validity of this approximation. In section 4 we stress how in the thermal leptogenesis scenario, when one starts with a vanishing initial abundance for the right handed neutrinos and assumes that no other interactions besides the Yukawa couplings can produce them, washout processes are essential to generate a lepton asymmetry. This is due to the fact that the lepton asymmetry produced at early times (mainly through scatterings mediated by off-shell N_1 and processes producing real N_1's) tends to be compensated by the opposite sign asymmetry produced at late times (when N_1 disappearance processes become the dominant source). The inclusion in the BE of CP violating asymmetries in scatterings is important to make this cancellation complete. In the presence of washouts, the cancellation can be partially avoided because washout processes erase slightly more efficiently the asymmetry produced at early times than the asymmetry produced at later times. However, when washouts are particularly weak the cancellation remains effective, and in this case the inclusion of CP violation in scatterings leads to a sizeable suppression of the final lepton asymmetry. In the strong washout regimes washout processes attain thermal equilibrium, thus erasing any dependence from the earlier conditions and in particular from the large asymmetries generated by the scattering processes. Since at lower temperatures the contributions from scatterings are highly suppressed and the lepton asymmetry is essentially generated through decays, for the strong washout regimes the inclusion of scattering CP asymmetries leaves the final results approximately unchanged.

2. The Boltzmann equations

We consider thermal leptogenesis scenarios with hierarchical heavy neutrinos N_i, i.e. with masses $M_{2,3} \gg M_1$. We will focus on the evolution of the lightest one, that in this section will be denoted simply by N, and we will ignore the possible contributions from the heavier neutrinos $N_{2,3}$ [22] except for their virtual effects in the CP violating asymmetries.

Let us first introduce some notation. We denote the thermally averaged rate for an initial state A to go into the final state B (summed over initial and final spin and gauge degrees of freedom) as:

$$\gamma_B^A \equiv \gamma(A \to B), \tag{2.1}$$

and the CP difference between the processes for particles and antiparticles as

$$\Delta\gamma_B^A \equiv \gamma_B^A - \gamma_{\bar{B}}^{\bar{A}}. \tag{2.2}$$

Particle densities are written in terms of the entropy density s, i.e. $Y_a \equiv n_a/s$ where n_a is the number density for the particle a. To simplify the expressions we rescale the densities Y_a by the equilibrium density Y_a^{eq} of the corresponding particle, defining $y_a \equiv Y_a/Y_a^{\mathrm{eq}}$, while the asymmetries of the rescaled densities are denoted[1] by $\Delta y_a \equiv y_a - y_{\bar{a}}$.

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[1]The notation adopted here differs from the notation used in [11, 14] in which the symbol y_a, rather than Δy_a, was used to denote the asymmetries of the rescaled densities.

The difference between a process and its time reversed, weighted by the densities of the initial state particles, is denoted as

$$[A \leftrightarrow B] \equiv \left(\prod_{i=1}^{n} y_{a_i} \right) \gamma_B^A - \left(\prod_{j=1}^{m} y_{b_j} \right) \gamma_A^B, \tag{2.3}$$

where the state A contains the particles $a_1, \ldots, a_n$ while the state B contains the particles $b_1, \ldots, b_m$. We will consider only processes in which at most one intermediate state heavy neutrino N can go on the mass shell, and in these cases a primed notation $\gamma_B'^A$ will refer to the rates with the resonant intermediate state (RIS) subtracted (and similarly for $[A \leftrightarrow B]'$). Accordingly, a process $A \to B$ of this kind will be divided into a RIS subtracted (off-shell) piece, and a second part corresponding to on-shell N exchange:

$$\gamma_B^A = \gamma_B'^A + \gamma_B^{\mathrm{os}\,A}. \tag{2.4}$$

In the simple case when only $2 \leftrightarrow 2$ scatterings are considered, the on-shell part is just

$$\gamma_B^{\mathrm{os}\,A} \equiv \gamma_N^A B_B^N, \tag{2.5}$$

where B_B^N denotes the branching ratio for N decays into the final state B. (As is discussed in section 2.2, the inclusion of $2 \leftrightarrow 3$ scatterings, and in general of processes of higher order in the couplings, implies that eq. (2.5) needs to be generalized.)

In the following two sections we will write down the BE for the evolution of the density of the heavy Majorana neutrinos N and of the asymmetry for a generic lepton flavor i. We will first consider in section 2.1 the leading terms involving the Yukawa couplings (generically denoted as λ) of the neutrino N to the Higgs boson H and to a light lepton doublet ℓ_i. In section 2.2 we will include the additional contributions arising from processes involving also the top Yukawa coupling h_t and the gauge interactions. Accordingly, the contributions to the evolution equation for the N density will be split in two parts:

$$\dot{Y}_N = \left(\dot{Y}_N \right)_I + \left(\dot{Y}_N \right)_{II}, \tag{2.6}$$

where we have introduced the notation $\dot{Y} \equiv szH\mathrm{d}Y/\mathrm{d}z$, with $z \equiv M_N/T$ and $H(z)$ being the Hubble rate at temperature T. In eq. (2.6) $\left(\dot{Y}_N \right)_I$ includes the contributions of terms up to $\mathcal{O}(\lambda^2)$, while $\left(\dot{Y}_N \right)_{II}$ includes the contributions up to $\mathcal{O}(\lambda^2 h_t^2)$ or $\mathcal{O}(\lambda^2 g^2)$, with g a generic gauge coupling constant.

The contributions to the evolution equation for the density of the lepton flavor i will also be split in different parts:

$$\dot{Y}_{L_i} = \left(\dot{Y}_{L_i} \right)_I + \left(\dot{Y}_{L_i} \right)_{II} + \left(\dot{Y}_{L_i} \right)_{\mathrm{sphal}}, \tag{2.7}$$

with $Y_{L_i} \equiv 2Y_{\ell_i} + Y_{e_i}$, where the factor 2 comes from summing over the densities of the two gauge degrees of freedom in ℓ_i, and the inclusion of the density Y_{e_i} for the right-handed lepton e_i is required because the L-conserving charged lepton Yukawas can transfer part of the asymmetry to the right handed degrees of freedom. In eq. (2.7) $\left(\dot{Y}_{L_i} \right)_I$ includes the

– 3 –

contributions of terms up to $\mathcal{O}(\lambda^4)$, while $\left(\dot{Y}_{L_i}\right)_{\text{II}}$ includes the contributions up to $\mathcal{O}(\lambda^4 h_t^2)$ or $\mathcal{O}(\lambda^4 g^2)$.

The last term $\left(\dot{Y}_{L_i}\right)_{\text{sphal}}$ represents the change in the lepton densities due to electroweak sphalerons, which are the only source of baryon number violation. Although the precise rates of sphaleron effects are hard to estimate, one knows that they are efficient below $T \simeq 10^{12}\,\text{GeV}$ and that they leave unchanged $B - L$. Moreover, sphalerons generate the same change in the baryon number of each generation, in fact one has that $(\dot{Y}_{\Delta L_i})_{\text{sphal}} = (\dot{Y}_{\Delta B})_{\text{sphal}}/3$, where $Y_{\Delta L_i} \equiv Y_{L_i} - Y_{\bar{L}_i}$ and $Y_{\Delta B}$ is the baryon asymmetry to entropy ratio. Hence, it is convenient to write directly an equation for the quantities $Y_{\Delta_i} \equiv Y_{\Delta B}/3 - Y_{\Delta L_i}$ which do not depend on the sphaleron rates. By subtracting from eq. (2.7) the analogous equation for $\dot{Y}_{\bar{L}_i}$ and by subtracting again the result from the equation that describes the evolution of the baryon asymmetry, $(\dot{Y}_{\Delta B})/3 = (\dot{Y}_{\Delta B})_{\text{sphal}}/3$, one obtains

$$\dot{Y}_{\Delta_i} = -\left(\dot{Y}_{\Delta L_i}\right)_I - \left(\dot{Y}_{\Delta L_i}\right)_{\text{II}}. \tag{2.8}$$

2.1 $1 \leftrightarrow 2$ and $2 \leftrightarrow 2$ processes

Let us briefly sketch the way in which the BE are obtained, by considering first the lepton (flavor) number violating $2 \leftrightarrow 2$ scatterings mediated by N exchange, that is $\ell_i H \leftrightarrow \ell_j H$ (with $j \neq i$) and $\ell_i H \leftrightarrow \bar{\ell}_j \bar{H}$. The BE describe the evolution of the particle densities at any given time, but since the Universe is expanding, the temperature is decreasing and the particle densities are changing, only processes that to a sufficiently good approximation can be described by (effective) contact interactions can be consistently included. However, the $2 \leftrightarrow 2$ scatterings we are considering can proceed through the exchange in the s-channel of an on-shell N, and in this case they are characterized by a time scale that can be comparable to the expansion rate of the Universe (at $T \sim M_1$). Therefore they cannot be approximated by a contact interaction, and must be treated with care. The usual way to deal with this is to separate the off-shell part of the scatterings from the on-shell piece, i.e.

$$\gamma_{\bar{\ell}_j \bar{H}}^{\ell_i H} \equiv \gamma'^{\ell_i H}_{\bar{\ell}_j \bar{H}} + \gamma^{\text{os}\,\ell_i H}_{\bar{\ell}_j \bar{H}}. \tag{2.9}$$

and similarly for $\ell_i H \leftrightarrow \ell_j H$. Then, at each given time, the evolution of the density of the lepton flavor L_i is determined by the following (instantaneous) reactions:

(i) $N \to \ell_i H$ decays occurring at a rate proportional to the density Y_N of the right handed neutrinos, and $\ell_i H \to N$ inverse decays;

(ii) off-shell $2 \leftrightarrow 2$ scatterings $\gamma'^{\bar{\ell}_j \bar{H}}_{\ell_i H}$ and $\gamma'^{\ell_j H}_{\ell_i H}$ involving only the exchange of virtual N's (in the first process N can be exchanged in both the s and t channels, while in the second process only in the s channel);

(iii) $2 \leftrightarrow 2$ scatterings $\gamma^{\bar{H}\bar{H}}_{\ell_i \ell_j}$ and $\gamma^{H\bar{H}}_{\ell_i \bar{\ell}_j}$ with N exchanged in the t and u-channels (in these cases no RIS can appear, and hence there are no on-shell contributions to be subtracted).

The corresponding BE then reads:

$$\left(\dot{Y}_{L_i}\right)_I = \left(\dot{Y}_{L_i}\right)_{1\leftrightarrow 2} + \left(\dot{Y}_{L_i}\right)_{2\leftrightarrow 2}^{\text{sub}} + \left(\dot{Y}_{L_i}\right)_{2\leftrightarrow 2}^{N_t},\tag{2.10}$$

where

$$\left(\dot{Y}_{L_i}\right)_{1\leftrightarrow 2} = [N \leftrightarrow \ell_i H],\tag{2.11}$$

$$\left(\dot{Y}_{L_i}\right)_{2\leftrightarrow 2}^{\text{sub}} = \sum_j [\bar{\ell}_j \bar{H} \leftrightarrow \ell_i H]' + \sum_{j\neq i}[\ell_j H \leftrightarrow \ell_i H]',\tag{2.12}$$

$$\left(\dot{Y}_{L_i}\right)_{2\leftrightarrow 2}^{N_t} = \sum_j \left\{[H\bar{H} \leftrightarrow \ell_i \bar{\ell}_j] + (1+\delta_{ij})[\bar{H}\bar{H} \leftrightarrow \ell_i \ell_j]\right\}.\tag{2.13}$$

It is important to remark that while eq. (2.11) contains processes of $\mathcal{O}(\lambda^2)$, both eqs. (2.12) and (2.13) contain only non-resonant scatterings, that are $\mathcal{O}(\lambda^4)$. However, while in a first approximation the contributions in eq. (2.13) may be neglected, the inclusion of the off-shell contributions of eq. (2.12) is mandatory. This is because the CP asymmetries of the subtracted rates are of the same order than the CP asymmetries of decays and inverse decays, and therefore neglecting them would yield inconsistent results. We can now subtract from eq. (2.10) the analogous equation for $Y_{\bar{L}_i}$ and write separately the source and washout contributions to $Y_{\Delta L_i} \equiv Y_{L_i} - Y_{\bar{L}_i}$ as:

$$\left(\dot{Y}_{\Delta L_i}\right)_I = \left(\dot{Y}_{\Delta L_i}\right)_I^s + \left(\dot{Y}_{\Delta L_i}\right)_I^w.\tag{2.14}$$

At the leading order, the source term receives contributions from the $1 \leftrightarrow 2$ decays and inverse decays in eq. (2.11) and from the off-shell parts of the $2 \leftrightarrow 2$ scatterings in eq. (2.12), while the CP asymmetries of the t- and u-channel processes in eq. (2.13), being of higher order in the couplings, can be neglected. We can then write the source term as:

$$\left(\dot{Y}_{\Delta L_i}\right)_I^s = \left(\dot{Y}_{\Delta L_i}\right)_{1\leftrightarrow 2}^s + \left(\dot{Y}_{\Delta L_i}\right)_{2\leftrightarrow 2}^{s\,\text{sub}},\tag{2.15}$$

where

$$\left(\dot{Y}_{\Delta L_i}\right)_{1\leftrightarrow 2}^s = (y_N + 1)\Delta\gamma_{\ell_i H}^N,\tag{2.16}$$

$$\left(\dot{Y}_{\Delta L_i}\right)_{2\leftrightarrow 2}^{s\,\text{sub}} = 2\left(\sum_j \Delta\gamma_{\ell_i H}'^{\bar{\ell}_j \bar{H}} + \sum_{j\neq i}\Delta\gamma_{\ell_i H}'^{\ell_j H}\right).\tag{2.17}$$

In order to proceed we now use an important relation that states that the CP asymmetries in the off-shell scatterings are, to leading order in the couplings, equal in magnitude and opposite in sign with respect to the CP asymmetries of the corresponding on-shell scatterings:

$$\Delta\gamma_B'^A \simeq -\Delta\gamma_B^{\text{os}\,A}.\tag{2.18}$$

To derive this relation, we first substitute the definition of the off-shell rates in eq. (2.4) to get $\Delta\gamma_B'^A = \Delta\gamma_B^A - \Delta\gamma_B^{\text{os}\,A}$ and then we use the fact that the CP asymmetry of any

process is always of higher order in the couplings with respect to the corresponding tree level process [24]. Thus the CP asymmetries $\Delta\gamma^{\bar{\ell}_j\bar{H}}_{\ell_i H}$ and $\Delta\gamma^{\ell_j H}_{\ell_i H}$ of the full $2\leftrightarrow 2$ scatterings are $\mathcal{O}(\lambda^6)$. On the other hand, the CP asymmetries of the on-shell pieces are of $\mathcal{O}(\lambda^4)$. This can be seen by writing them in terms of eq. (2.5) to obtain

$$\Delta\gamma^{\mathrm{os}A}_B = \Delta\gamma^A_N\, B^N_B + \gamma^A_N \frac{\Delta\gamma^N_B}{\gamma_{\mathrm{tot}}} \simeq B^N_A \Delta\gamma^N_B - B^N_B \Delta\gamma^N_A. \tag{2.19}$$

In eq. (2.19) γ_{tot} is the total N decay rate, and we have used $\Delta\gamma^A_N = \Delta\gamma^N_A = -\Delta\gamma^N_{\bar{A}}$ where the first equality follows from the CPT relation $\gamma(A\to B) = \gamma(\bar{B}\to\bar{A})$, and we have approximated at leading order $B^N_{\bar{A}} \simeq B^N_A$. This shows that $\Delta\gamma^{\mathrm{os}A}_B$ (and hence $\Delta\gamma'^A_B$) is of the same order in the couplings as the CP asymmetry in decays $\Delta\gamma^N_A$ (that is $\mathcal{O}(\lambda^4)$). Therefore, up to $\mathcal{O}(\lambda^6)$ corrections, the contributions of the off-shell rates in eq. (2.17) can be written as

$$\left(\dot{Y}_{\Delta L_i}\right)^{s\,\mathrm{sub}}_{2\leftrightarrow 2} \simeq -2\Delta\gamma^N_{\ell_i H} \sum_j \left(B^N_{\ell_j H} + B^N_{\bar{\ell}_j\bar{H}}\right). \tag{2.20}$$

Here $B^N_{\ell_j H}$ represents the branching ratio for the decay $N\to\ell_j H$. At the order in the couplings we are working here $\sum_j\left(B^N_{\ell_j H} + B^N_{\bar{\ell}_j\bar{H}}\right)\simeq 1$ and therefore the r.h.s. of eq. (2.20) can be further simplified to $-2\Delta\gamma^N_{\ell_i H}$. After summing up the two contributions (2.16) and (2.20), the source term $\left(\dot{Y}_{L_i}\right)_I$ in eq. (2.15) becomes proportional to $y_N - 1$. This is in agreement with the general condition that no asymmetry can be generated in thermal equilibrium, and constitutes a check that, at this order, all the relevant contributions to the source term of the BE have been included.

The washout term $\left(\dot{Y}_{\Delta L_i}\right)^w_I$ in eq. (2.14) contains the terms proportional to the light particle asymmetries, and is the sum of three different contributions obtained by subtracting from eqs. (2.11)–(2.13) the corresponding equations for $Y_{\bar{L}_i}$. It can thus be written as

$$\left(\dot{Y}_{\Delta L_i}\right)^w_I = \left(\dot{Y}_{\Delta L_i}\right)^w_{1\leftrightarrow 2} + \left(\dot{Y}_{\Delta L_i}\right)^{w,\mathrm{sub}}_{2\leftrightarrow 2} + \left(\dot{Y}_{\Delta L_i}\right)^{w,Nt}_{2\leftrightarrow 2}. \tag{2.21}$$

After linearizing in the CP asymmetries and in the asymmetries of the normalized densities, these contributions read:

$$\left(\dot{Y}_{\Delta L_i}\right)^w_{1\leftrightarrow 2} = -(\Delta y_{\ell_i} + \Delta y_H)\gamma^{\ell_i H}_N, \tag{2.22}$$

$$\left(\dot{Y}_{\Delta L_i}\right)^{w,\mathrm{sub}}_{2\leftrightarrow 2} = -\sum_j\left[(\Delta y_{\ell_i} + \Delta y_H)\left(\gamma'^{\ell_i H}_{\bar{\ell}_j\bar{H}} + \gamma'^{\ell_i H}_{\ell_j H}\right) + (\Delta y_{\ell_j} + \Delta y_H)\left(\gamma'^{\ell_i H}_{\bar{\ell}_j\bar{H}} - \gamma'^{\ell_i H}_{\ell_j H}\right)\right],$$

$$\left(\dot{Y}_{\Delta L_i}\right)^{w,N_t}_{2\leftrightarrow 2} = -\sum_j\left[(1+\delta_{ij})(\Delta y_{\ell_i} + \Delta y_{\ell_j} + 2\Delta y_H)\gamma^{\ell_i\ell_j}_{\bar{H}\bar{H}} + (\Delta y_{\ell_i} - \Delta y_{\ell_j})\gamma^{\ell_i\bar{\ell}_j}_{\bar{H}H}\right].$$

One can estimate the off-shell parts in eq. (2.23) by introducing a subtracted propagator for N, or alternatively eliminating the γ' by means eq. (2.4).

Finally, to compute the lepton asymmetry we also need to solve for the evolution of the heavy neutrino density y_N that appears in the source term eq. (2.16). To the leading

order in the couplings, the corresponding BE reads

$$\left(\dot{Y}_N\right)_I = \sum_j \left\{[\ell_j H \leftrightarrow N] + [\bar{\ell}_j \bar{H} \leftrightarrow N]\right\} \simeq -(y_N - 1)\gamma_D^{N\to2}, \qquad (2.23)$$

where $\gamma_D^{N\to2} = \sum_j (\gamma_{\ell_j H}^N + \gamma_{\bar{\ell}_j \bar{H}}^N)$ is the thermally averaged two body N decay rate, and we have approximated the leptons and Higgs particle densities with their equilibrium values.

2.2 $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ processes

We are now ready to generalize the above procedure to include processes involving the Higgs Yukawa coupling h_t to the right handed top quark, which is sizeable. Processes involving gauge bosons can be included in an entirely similar way and will be mentioned later.

The inclusion of $\Delta L = 1$ processes like $N \leftrightarrow \ell_i \bar{Q} t$ decays and inverse decays (where Q is the left-handed quark doublet and t the right-handed top singlet) and scatterings mediated by Higgs exchange like $N\ell_i \leftrightarrow Q\bar{t}$, follows along lines analogous to those presented in the previous section. For the evolution of the heavy neutrino density we obtain

$$\left(\dot{Y}_N\right)_{II} = -(y_N - 1)\left[\gamma_D^{N\to3} + \gamma_{top}^{2\leftrightarrow2}\right], \qquad (2.24)$$

where

$$\gamma_D^{N\to3} \equiv \sum_j (\gamma_{\ell_j \bar{Q} t}^N + \gamma_{\bar{\ell}_j Q \bar{t}}^N) \qquad (2.25)$$

is the contribution from decays into three body final states, and the contribution from Higgs mediated scatterings is

$$\gamma_{top}^{2\leftrightarrow2} = \sum_j \left(\gamma_{Q\bar{t}}^{N\ell_j} + \gamma_{\bar{Q}t}^{N\bar{\ell}_j} + \gamma_{\ell_j t}^{NQ} + \gamma_{\bar{\ell}_j \bar{t}}^{N\bar{Q}} + \gamma_{\bar{\ell}_j Q}^{Nt} + \gamma_{\ell_j \bar{Q}}^{N\bar{t}}\right), \qquad (2.26)$$

where the first two terms in the sum correspond to the contributions from s-channel Higgs exchange, while the other four terms (that at leading order are all equal) correspond to the contribution from t and u channel Higgs exchange.

Regarding the evolution of the lepton asymmetries, the derivation of the BE is more delicate because, besides the inclusion of the CP violating asymmetries in $1 \leftrightarrow 3$ decays like $N \leftrightarrow \ell_i \bar{Q} t$ and in $2 \leftrightarrow 2$ scatterings like $N\ell_i \leftrightarrow Q\bar{t}$, the asymmetries of various off-shell $2 \leftrightarrow 3$ scatterings, that contribute to the source term at the same order in the couplings, should also be included. Accordingly, the term $\left(\dot{Y}_{L_i}\right)_{II}$ in eq. (2.7) can be written as

$$\left(\dot{Y}_{L_i}\right)_{II} = \left(\dot{Y}_{L_i}\right)_{\substack{1\leftrightarrow3 \\ 2\leftrightarrow2}} + \left(\dot{Y}_{L_i}\right)_{2\leftrightarrow3}^{sub} + \left(\dot{Y}_{L_i}\right)_{2\leftrightarrow3}^{N_t} \qquad (2.27)$$

with

$$\left(\dot{Y}_{L_i}\right)_{\substack{1\leftrightarrow 3 \\ 2\leftrightarrow 2}} = [N \leftrightarrow \ell_i \bar{Q}t] + [Q\bar{t} \leftrightarrow N\ell_i] + [N\bar{t} \leftrightarrow \bar{Q}\ell_i] + [NQ \leftrightarrow t\ell_i]\,; \tag{2.28}$$

$$\begin{aligned}
\left(\dot{Y}_{L_i}\right)_{2\leftrightarrow 3}^{\text{sub}} &= \sum_{j\neq i} \left\{ [\ell_j H \leftrightarrow \bar{Q}t\ell_i]' + [\ell_j HQ \leftrightarrow t\ell_i]' + [\ell_j H\bar{t} \leftrightarrow \bar{Q}\ell_i]' \right. \\
&\quad \left. + [\ell_j \bar{Q}t \leftrightarrow \ell_i H]' + [\ell_j \bar{Q} \leftrightarrow \ell_i H\bar{t}]' + [\ell_j t \leftrightarrow \ell_i HQ]' \right\} \\
&\quad + \sum_j \left\{ [\bar{\ell}_j \bar{H} \leftrightarrow \ell_i \bar{Q}t]' + [\bar{\ell}_j \bar{H}Q \leftrightarrow t\ell_i]' + [\bar{\ell}_j \bar{H}\bar{t} \leftrightarrow \bar{Q}\ell_i]' + [\bar{\ell}_j Q\bar{t} \leftrightarrow \ell_i H]' \right. \\
&\quad \left. + [\bar{\ell}_j Q \leftrightarrow \ell_i Ht]' + [\bar{\ell}_j \bar{t} \leftrightarrow \ell_i H\bar{Q}]' + [\bar{Q}t \leftrightarrow \ell_i H\bar{\ell}_j]' + [Q\bar{t} \leftrightarrow \ell_i \bar{H}\bar{\ell}_j]' \right\} \\
&\quad + \sum_j (1+\delta_{ij})[Q\bar{t} \leftrightarrow \ell_i H\ell_j]'\,; \tag{2.29}
\end{aligned}$$

$$\begin{aligned}
\left(\dot{Y}_{L_i}\right)_{2\leftrightarrow 3}^{N_t} &= \sum_{j\neq i} \left\{ [\ell_j Q\bar{t} \leftrightarrow \ell_i \bar{H}] + [\ell_j \bar{H} \leftrightarrow \ell_i Q\bar{t}] + [\ell_j \bar{H}t \leftrightarrow \ell_i Q] + [\ell_j \bar{H}\bar{Q} \leftrightarrow \ell_i \bar{t}] \right\} \\
&\quad + \sum_j \left\{ [Q\bar{t}H \leftrightarrow \bar{\ell}_j \ell_i] + [\bar{Q}t\bar{H} \leftrightarrow \bar{\ell}_j \ell_i] + [\bar{Q}\bar{H} \leftrightarrow \bar{\ell}_j \ell_i \bar{t}] + [t\bar{H} \leftrightarrow \bar{\ell}_j \ell_i \bar{Q}] \right\} \\
&\quad + \sum_j (1+\delta_{ij}) \left\{ [\bar{t}\bar{H} \leftrightarrow \ell_i \bar{Q}\ell_j] + [Q\bar{t}\bar{H} \leftrightarrow \ell_i \ell_j] + [Q\bar{H} \leftrightarrow \ell_i t\ell_j] \right\}\,. \tag{2.30}
\end{aligned}$$

As in the previous section, the asymmetries of the off-shell $2 \leftrightarrow 3$ rates in eq. (2.29) can be estimated by relating them to the asymmetries of the corresponding on-shell parts. However, for $2 \leftrightarrow 3$ scatterings the definition of the on-shell part is more subtle, because after a real N is produced in a collision, it has a certain probability to scatter before decaying. Let us consider for example the process $\ell_j HQ \to t\ell_i$. The contribution to this process from the exchange of an on-shell N corresponds to the production process $\ell_j H \to N$ followed by the scattering $N + Q \to t\ell_i$ mediated by a Higgs in the t channel. Processes of this kind can generally be written as $AX \to Y$ where A denotes a possible state to which N can decay. The corresponding on-shell rate then is

$$\gamma^{\text{os}\,AX}_Y = \gamma^A_N P^{NX}_Y, \tag{2.31}$$

where we have introduced the quantity P^{NX}_Y that is the probability that the heavy neutrino N will scatter with X to produce Y. Processes in which the on-shell N can disappear only by decaying (as for example $\ell_j H \leftrightarrow \ell_i \bar{Q}t$ or $\ell_j t \leftrightarrow \ell_i HQ$) can generally be written as $A \to B$ or as $X \to BY$ where both A and B denote possible final states for N decays. The corresponding on-shell rates are

$$\gamma^{\text{os}\,A}_B = \gamma^A_N P^N_B, \tag{2.32}$$

$$\gamma^{\text{os}\,X}_{BY} = \gamma^X_{NY} P^N_B. \tag{2.33}$$

Note that because of the fact that in the dense plasma N can suffer inelastic scatterings before decaying as is described by eq. (2.31), the quantities P^N_B in eqs. (2.32) and (2.33) differ from the usual notion of branching ratios at zero temperature. In particular, scattering rates should also be included in normalizing properly the decay probabilities, and

the quantities P_b^a then will denote the general probabilities that the heavy neutrino N contained in state a ends up producing state b. In the case under discussion, we have for example

$$
\begin{aligned}
P_{\ell_i H}^N &= \frac{\gamma_{\ell_i H}^N}{\gamma_{\text{all}}}, \qquad P_{\ell_i \bar{Q} t}^N = \frac{\gamma_{\ell_i \bar{Q} t}^N}{\gamma_{\text{all}}}, \\
P_{Q\bar{t}}^{N\ell_i} &= \frac{\gamma_{Q\bar{t}}^{N\ell_i}}{\gamma_{\text{all}}}, \qquad P_{t\ell_i}^{NQ} = \frac{\gamma_{t\ell_i}^{NQ}}{\gamma_{\text{all}}}, \qquad P_{\bar{Q}\ell_i}^{N\bar{t}} = \frac{\gamma_{\bar{Q}\ell_i}^{N\bar{t}}}{\gamma_{\text{all}}},
\end{aligned}
\tag{2.34}
$$

with similar definitions for the probabilities of the CP conjugate processes. The probabilities are normalized in terms of the sum of all the rates, that reads

$$
\gamma_{\text{all}} = \sum_i (\gamma_{\ell_i H}^N + \gamma_{\bar{\ell}_i \bar{H}}^N + \gamma_{\ell_i \bar{Q} t}^N + \gamma_{\bar{\ell}_i Q \bar{t}}^N + \gamma_{Q\bar{t}}^{N\ell_i} + \gamma_{\bar{Q}t}^{N\bar{\ell}_i} + \gamma_{\bar{\ell}_i \bar{t}}^{NQ} + \gamma_{\ell_i t}^{N\bar{Q}} + \gamma_{\ell_i \bar{Q}}^{Nt} + \gamma_{\bar{\ell}_i Q}^{Nt}). \tag{2.35}
$$

To the order in the Yukawa couplings that we are considering, the unitarity condition for the sum of the branching ratios of N into all possible final states $\sum_Y B_Y^N = 1$ is then generalized to $\sum_{X,Y} P_Y^{NX} = 1$. That is, the probabilities for all the possible ways through which N can disappear add up to unity.

To include the new sources of CP asymmetries, we now need to subtract from eqs. (2.28)–(2.30) the analogous equation for $Y_{\bar{L}_i}$. We obtain

$$
(\dot{Y}_{\Delta L_i})_{\text{II}}^s = (\dot{Y}_{\Delta L_i})_{\substack{1\leftrightarrow 3 \\ 2\leftrightarrow 2}}^s + \left(\dot{Y}_{\Delta L_i}\right)_{2\leftrightarrow 3}^{s,\,\text{sub}}, \tag{2.36}
$$

where we have neglected the CP asymmetries of the $2 \leftrightarrow 3$ processes with N exchanged in the t-channel since they are of higher order in the couplings. For the first term in the r.h.s. of eq. (2.36) we have

$$
(\dot{Y}_{\Delta L_i})_{\substack{1\leftrightarrow 3 \\ 2\leftrightarrow 2}}^s = (y_N + 1) \left[\Delta\gamma_{\ell_i \bar{Q} t}^N + \Delta\gamma_{\ell_i \bar{Q}}^{N\bar{t}} + \Delta\gamma_{\ell_i t}^{NQ} - \Delta\gamma_{Q\bar{t}}^{N\ell_i} \right]. \tag{2.37}
$$

After eliminating the subtracted rates by writing their CP asymmetries as minus the CP asymmetries of the on-shell rates and keeping terms up to $\mathcal{O}(\lambda^4 h_t^2)$, we obtain for the second term in eq. (2.36)

$$
\left(\dot{Y}_{\Delta L_i}\right)_{2\leftrightarrow 3}^{s,\,\text{sub}} = -2\,\Delta\gamma_{\ell_i H}^N \left[1 - \sum_j \left(P_{\ell_j H}^N + P_{\bar{\ell}_j \bar{H}}^N \right) \right] - 2\left[\Delta\gamma_{\ell_i \bar{Q} t}^N + \Delta\gamma_{\ell_i \bar{Q}}^{N\bar{t}} + \Delta\gamma_{\ell_i t}^{NQ} - \Delta\gamma_{Q\bar{t}}^{N\ell_i} \right]. \tag{2.38}
$$

Note that at order $\mathcal{O}(\lambda^4 h_t^2)$ eq. (2.20) reads

$$
\left(\dot{Y}_{\Delta L_i}\right)_{2\leftrightarrow 2}^{s\,\text{sub}} = -2\Delta\gamma_{\ell_i H}^N \sum_j \left(P_{\ell_j H}^N + P_{\bar{\ell}_j \bar{H}}^N \right), \tag{2.39}
$$

where the sum of the probabilities $\sum_j \left(P_{\ell_j H}^N + P_{\bar{\ell}_j \bar{H}}^N \right)$ is not unity. However, the first term in eq. (2.38) conspires with eq. (2.39) to yield the correct behavior for the source term involving $\Delta\gamma_{\ell_i H}^N$, that turns out again to be proportional to $(y_N - 1)$. By summing up

eqs. (2.16), (2.37), (2.38) and (2.39) we obtain the final expression for the source term that holds at $\mathcal{O}(\lambda^4 h_t^2)$:

$$(\dot{Y}_{\Delta L_i})^s_{I+II} = (y_N - 1)\left[\Delta\gamma^N_{\ell_i H} + \Delta\gamma^N_{\ell_i \bar{Q}t} + \Delta\gamma^{N\bar{t}}_{\ell_i \bar{Q}} + \Delta\gamma^{NQ}_{\ell_i t} - \Delta\gamma^{N\ell_i}_{Q\bar{t}}\right]. \tag{2.40}$$

Regarding the washouts, the contributions from eqs. (2.28)–(2.30) after subtracting the analogous equations for $\dot{Y}_{\bar{L}_i}$ can be written as

$$\left(\dot{Y}_{\Delta L_i}\right)^w_{II} = \left(\dot{Y}_{\Delta L_i}\right)^w_{\substack{1\leftrightarrow3\\2\leftrightarrow2}} + \left(\dot{Y}_{\Delta L_i}\right)^{w,\,\mathrm{sub}}_{2\leftrightarrow3} + \left(\dot{Y}_{\Delta L_i}\right)^{w,\,N_t}_{2\leftrightarrow3}, \tag{2.41}$$

where

$$\left(\dot{Y}_{\Delta L_i}\right)^w_{\substack{1\leftrightarrow3\\2\leftrightarrow2}} = \Big[(\Delta y_Q - \Delta y_t - \Delta y_{\ell_i})\gamma^N_{\ell_i\bar{Q}t} + (\Delta y_Q - \Delta y_t - y_N\Delta y_{\ell_i})\gamma^{Q\bar{t}}_{N\ell_i}$$
$$+ (\Delta y_Q - y_N\Delta y_t - \Delta y_{\ell_i})\gamma^{N\bar{t}}_{\bar{Q}\ell_i} + (y_N\Delta y_Q - \Delta y_t - \Delta y_{\ell_i})\gamma^{NQ}_{t\ell_i}\Big]; \tag{2.42}$$

$$\left(\dot{Y}_{\Delta L_i}\right)^{w,\,\mathrm{sub}}_{2\leftrightarrow3} = \sum_{j\neq i}\Big[(\Delta y_{\ell_j} + \Delta y_Q - \Delta y_t + \Delta y_H - \Delta y_{\ell_i})\left(\gamma'^{\ell_j H}_{\bar{Q}t\ell_i} + \gamma'^{\ell_j HQ}_{t\ell_i} + \gamma'^{\ell_j H\bar{t}}_{\bar{Q}\ell_i}\right)$$
$$+ (\Delta y_{\ell_j} - \Delta y_Q + \Delta y_t - \Delta y_H - \Delta y_{\ell_i})\left(\gamma'^{\ell_j\bar{Q}t}_{H\ell_i} + \gamma'^{\ell_j\bar{Q}}_{H\bar{t}\ell_i} + \gamma'^{\ell_j t}_{QH\ell_i}\right)\Big]$$
$$+ \sum_j\Big[(-\Delta y_{\ell_j} + \Delta y_Q - \Delta y_t - \Delta y_H - \Delta y_{\ell_i})\times$$
$$\left(\gamma'^{\bar{\ell}_j\bar{H}}_{\bar{Q}t\ell_i} + \gamma'^{\bar{\ell}_j\bar{H}Q}_{t\ell_i} + \gamma'^{\bar{\ell}_j\bar{H}\bar{t}}_{\bar{Q}\ell_i} + \gamma'^{\bar{\ell}_j Q\bar{t}}_{H\ell_i} + \gamma'^{\bar{\ell}_j Q}_{tH\ell_i} + \gamma'^{\bar{\ell}_j\bar{t}}_{\bar{Q}H\ell_i} + (1+\delta_{ij})\gamma'^{Q\bar{t}}_{\ell_j H\ell_i}\right)$$
$$+ (\Delta y_{\ell_j} - \Delta y_Q + \Delta y_t - \Delta y_H - \Delta y_{\ell_i})\gamma'^{\bar{Q}t}_{\bar{\ell}_j H\ell_i}$$
$$+ (\Delta y_{\ell_j} + \Delta y_Q - \Delta y_t + \Delta y_H - \Delta y_{\ell_i})\gamma'^{Q\bar{t}}_{\bar{\ell}_j\bar{H}\ell_i}\Big]; \tag{2.43}$$

$$\left(\dot{Y}_{\Delta L_i}\right)^{w,\,N_t}_{2\leftrightarrow3} = \sum_{j\neq i}\Big[(\Delta y_{\ell_j} + \Delta y_Q - \Delta y_t + \Delta y_H - \Delta y_{\ell_i})\gamma^{\ell_j Q\bar{t}}_{\bar{H}\ell_i} \tag{2.44}$$
$$+ (\Delta y_{\ell_j} - \Delta y_Q + \Delta y_t - \Delta y_H - \Delta y_{\ell_i})(\gamma^{\ell_j\bar{H}}_{Q\bar{t}\ell_i} + \gamma^{\ell_j\bar{H}t}_{Q\ell_i} + \gamma^{\ell_j\bar{H}\bar{Q}}_{\bar{t}\ell_i})\Big]$$
$$+ \sum_j\Big[(\Delta y_{\ell_j} + \Delta y_Q - \Delta y_t + \Delta y_H - \Delta y_{\ell_i})\gamma^{Q\bar{t}H}_{\bar{\ell}_j\ell_i}$$
$$+ (\Delta y_{\ell_j} + \Delta y_Q + \Delta y_t - \Delta y_H - \Delta y_{\ell_i})\gamma^{t\bar{H}}_{\bar{\ell}_j\bar{Q}\ell_i}$$
$$+ (\Delta y_{\ell_j} - \Delta y_Q + \Delta y_t - \Delta y_H - \Delta y_{\ell_i})\left(\gamma^{\bar{Q}t\bar{H}}_{\bar{\ell}_j\ell_i} + \gamma^{\bar{Q}\bar{H}}_{\bar{\ell}_j\bar{t}\ell_i}\right)\Big]$$
$$+ \sum_j\Big\{(1+\delta_{ij})(-\Delta y_{\ell_j} + \Delta y_Q - \Delta y_t - \Delta y_H - \Delta y_{\ell_i})\left(\gamma^{\bar{t}\bar{H}}_{\ell_j\bar{Q}\ell_j} + \gamma^{Q\bar{t}\bar{H}}_{\ell_j\ell_j} + \gamma^{Q\bar{H}}_{\ell_j t\ell_j}\right)\Big\}. $$

We can now summarize (and slightly generalize) the procedure for writing the BE for leptogenesis. Ignoring processes with more than one N, one can write them as

$$\dot{Y}_N = -(y_N - 1)\sum_{A,B}\gamma^{NA}_B, \quad (\dot{Y}_{\Delta L_i})^s = (y_N - 1)\sum_{A,B}[L_i(B) - L_i(A)]\,\Delta\gamma^{NA}_B, \tag{2.45}$$

$$(\dot{Y}_{\Delta L_i})^w = \sum_{A,B}[L_i(B) - L_i(A)]\left(y_N^{n_A}\sum_{a_i}\Delta y_{a_i} - y_N^{n_B}\sum_{b_i}\Delta y_{b_i}\right)\gamma'^A_B, \tag{2.46}$$

where in the first two equations the states A and B contain only light standard model particles, while in the last one for the case in which no on-shell intermediate N is allowed in the process $A \to B$, one has simply $\gamma_B'^A = \gamma_B^A$. In these equations $L_i(A)$ denote the i lepton flavor number of state A while $n_A = 0$ or 1 counts the number of N's contained in state A (and similarly for $L_i(B)$ and n_B). To avoid double counting in both equations the sums are restricted to $L_i(B) > 0$ and $L_i(B) > L_i(A)$, where the first restriction avoids double counting the CP conjugate processes with $\bar{\ell}_i$ in the final states, and the second restriction avoids double counting the time reversed processes where the number of ℓ_i in the initial state is larger than in the final state. In the equation (2.46) for the washouts the a_i's and b_i's denote all the particle species with non-vanishing asymmetries contained respectively in states A and B. Let us note that in the cases we have been discussing all the CP violating asymmetries appearing in eq. (2.45) have $L_i(B) - L_i(A) = 1$, and thus the expression for the source term can be accordingly simplified. However, this is not true for the washout term since some processes with $L_i(B) - L_i(A) = 2$ also contribute.

Several contributions that we have included for consistency and for completeness, for practical purposes can be neglected without affecting sizably the numerical results. This is the case for example for all the non-resonant $2 \leftrightarrow 3$ washout terms. Also the inclusion of N decays into three body final states has been carried out mainly for the sake of completeness, that is to account for all the processes of the same order in the couplings, and also to incorporate consistently those $2 \leftrightarrow 3$ scatterings for which the on-shell piece involves precisely a $1 \to 3$ decay, as for example $\ell_j H \to N \to \ell_i \bar{Q} t$. However, the overall impact on quantitative results of the three-body decay CP asymmetry $\Delta\gamma^N_{\ell_i \bar{Q} t}$ (as well as the contribution to the washouts of $3 \to 1$ inverse decays) are rather small. This is because while e.g. $\Delta\gamma^N_{\ell_i \bar{Q} t}$ involves the same CP violating phase as $\Delta\gamma^N_{\ell_i H}$, it also has a significant suppression factor arising from three body phase space. The explicit expression for the three body decay rate is presented in the appendix.

Following the same procedure outlined above, it is possible to include in the BE other relevant processes, such as those involving the gauge bosons [7, 9]. With all the subdominant terms neglected and with the effects of the gauge bosons included, the simplified expression of the BE for the evolution of Y_N reads:

$$\dot{Y}_N = -(y_N - 1) \left[\gamma_D^{N \to 2} + \gamma_{\text{top}}^{2 \leftrightarrow 2} + \gamma_A^{2 \leftrightarrow 2} \right], \tag{2.47}$$

where the term involving the gauge bosons is defined as

$$\gamma_A^{2 \leftrightarrow 2} = \sum_j \left(\gamma_{A\bar{H}}^{N\ell_j} + \gamma_{AH}^{N\bar{\ell}_j} + \gamma_{A\ell_j}^{N\bar{H}} + \gamma_{A\bar{\ell}_j}^{NH} + \gamma_{\ell_j H}^{NA} + \gamma_{\bar{\ell}_j \bar{H}}^{NA} \right), \tag{2.48}$$

where $A = W_i$ or B for SU(2) and U(1) bosons respectively, and a sum over all the gauge boson degrees of freedom is understood. In eq. (2.47) (as well as in eq. (2.50) below) we have neglected three-body decays like $\gamma_{AH\ell_i}^N$ involving the gauge bosons because, similarly to the three-body decays involving the top quarks, they are suppressed by phase space factors and give negligible contributions, and we have also neglected the contributions to the washouts from $2 \leftrightarrow 3$ processes. The simplified expression for the evolution equation

for the charge $Y_{\Delta_i} = Y_{\Delta B}/3 - Y_{\Delta L_i}$ that is conserved by sphaleron interactions, is:

$$\dot{Y}_{\Delta_i} = \left(\dot{Y}_{\Delta_i}\right)^s + \left(\dot{Y}_{\Delta_i}\right)^w, \tag{2.49}$$

with

$$\left(\dot{Y}_{\Delta_i}\right)^s = -(y_N - 1)\left[\Delta\gamma_{\ell_i H}^N + \Delta\gamma_{\ell_i \bar{Q}}^{N\bar{t}} + \Delta\gamma_{\ell_i t}^{NQ} - \Delta\gamma_{Q\bar{t}}^{N\ell_i} - \Delta\gamma_{A\bar{H}}^{N\ell_i} + \Delta\gamma_{A\ell_i}^{N\bar{H}} + \Delta\gamma_{\ell_i H}^{NA}\right]. \tag{2.50}$$

$$\begin{aligned}
\left(\dot{Y}_{\Delta_i}\right)^w &= (\Delta y_{\ell_i} + \Delta y_H)\gamma_N^{\ell_i H} \\
&\quad + \sum_j \left[(\Delta y_{\ell_i} + \Delta y_H)\left(\gamma'^{\ell_i H}_{\bar{\ell}_j \bar{H}} + \gamma'^{\ell_i H}_{\ell_j H}\right) + (\Delta y_{\ell_j} + \Delta y_H)\left(\gamma'^{\ell_i H}_{\bar{\ell}_j \bar{H}} - \gamma'^{\ell_i H}_{\ell_j H}\right)\right] \\
&\quad + \sum_j \left[(1 + \delta_{ij})(\Delta y_{\ell_i} + \Delta y_{\ell_j} + 2\Delta y_H)\gamma_{\bar{H}\bar{H}}^{\ell_i \ell_j} + (\Delta y_{\ell_i} - \Delta y_{\ell_j})\gamma_{\bar{H}H}^{\ell_i \bar{\ell}_j}\right] \\
&\quad + (y_N \Delta y_{\ell_i} - \Delta y_Q + \Delta y_t)\gamma_{Q\bar{t}}^{N\ell_i} - (y_N \Delta y_Q - \Delta y_t - \Delta y_{\ell_i})\gamma_{t\ell_i}^{NQ} \\
&\quad - (-y_N \Delta y_t + \Delta y_Q - \Delta y_{\ell_i})\gamma_{\bar{Q}\ell_i}^{N\bar{t}} - (y_N \Delta y_{\ell_i} + \Delta y_H)\gamma_{A\bar{H}}^{N\ell_i} \\
&\quad + (y_N \Delta y_H + \Delta y_{\ell_i})\gamma_{A\bar{\ell}_i}^{NH} + (\Delta y_H + \Delta y_{\ell_i})\gamma_{H\ell_i}^{NA}. \tag{2.51}
\end{aligned}$$

Before concluding this section one more remark is in order. To solve the set of coupled BE for the different lepton flavors it is necessary to express all the normalized particle asymmetries Δy_a and Δy_H in terms of the relevant charge asymmetries Y_{Δ_i} (and use the relation $\Delta y_Q - \Delta y_t = \Delta y_H/2$). In doing so one has to take into account the constraints on chemical potentials enforced by the reactions that, in the particular temperature range in which leptogenesis occurs, are faster than the Universe expansion [11, 25]. Although we have written the equations for a generic asymmetry Y_{Δ_i} of one of the three flavors, it is important to keep in mind that when no lepton Yukawa couplings are in equilibrium ($T > 10^{12}\,\text{GeV}$), one has just an effective one-flavor equation for the asymmetry in the lepton ℓ coupled to the lightest heavy neutrino N. When just the τ Yukawa coupling is in equilibrium ($10^9\,\text{GeV} < T < 10^{12}\,\text{GeV}$) two flavor components are relevant, ℓ_τ and ℓ_a, where ℓ_a is the component of the lepton doublet produced in N decays which is orthogonal to ℓ_τ [14]. When also the μ Yukawa coupling is in equilibrium ($T < 10^9\,\text{GeV}$) the three flavor components of ℓ are completely projected out by the Yukawa interactions, and the full set of equations for the three Y_{Δ_i}, with $i = e$, μ, τ, is needed.

3. The CP asymmetry in scattering processes

We will now study the CP asymmetries in $2 \rightarrow 2$ scattering processes involving the top quark and with a Higgs exchanged in the s or in the t channels. These asymmetries were considered previously in [7] and [16] where some arguments were given in support of an approximate equality between the scatterings and the decay asymmetries, as for example:

$$\frac{\Delta\gamma_{N\ell_i}^{Q\bar{t}}}{\gamma_{N\ell_i}^{Q\bar{t}}} \simeq \frac{\Delta\gamma_{\ell_i H}^N}{\gamma_{\ell_i H}^N}. \tag{3.1}$$

In this section we will explore the validity of this kind of approximations.

The Lagrangian for the Yukawa interactions relevant for the computation of the associated CP asymmetries, written in the mass eigenstate basis of the heavy neutrinos $N_\alpha(\alpha = 1, 2, 3)$, of the charged leptons $(i = e, \mu, \tau)$ and of the quarks, reads:

$$\mathcal{L}_Y = -\lambda_{i\alpha}\, \overline{N}_\alpha \ell_i\, \widetilde{H}^\dagger - h_t\, \overline{Q} t\, \widetilde{H} + h.c. \,. \tag{3.2}$$

Here $H = (H^+, H^0)^T$ is the Higgs field, with $\widetilde{H} = i\tau_2 H^*$. Like in the usual case of the CP asymmetries in N decays, the CP asymmetries in scattering processes arise from the interference between the tree level and one loop amplitudes. They can be computed by explicit evaluation of the corresponding loop integrals, or just by using the Cutkowski rules that give directly the absorptive part of the Feynman diagrams.

The vertex contribution to the CP asymmetry in the scattering $Q\bar{t} \leftrightarrow N_\alpha \ell_i$ with the Higgs exchanged in the s-channel arises from the interference between the diagrams in figure $1(a)$ and $1(b)$. Expressed in terms of the CP difference between the squared invariant amplitudes $|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2$ this contribution reads

$$\left[|\mathcal{M}(Q\bar{t} \to N_\alpha \ell_i)|^2 - |\bar{\mathcal{M}}|^2\right](vertex) = -\frac{6}{\pi} \frac{p_1 \cdot p_2 \; p_1' \cdot p_2'}{s^2} |h_t|^2 \times$$

$$\sum_\beta \mathrm{Im}[\lambda_{i\alpha}\lambda_{i\beta}^*(\lambda^\dagger\lambda)_{\beta\alpha}] \frac{M_\alpha M_\beta}{s - M_\alpha^2} \left\{ \left[1 - \frac{M_\alpha^2 + M_\beta^2 - s}{M_\alpha^2 - s} \ln\left(\frac{|M_\alpha^2 + M_\beta^2 - s|}{M_\beta^2}\right)\right] \right.$$

$$\left. - \theta(s - M_\beta^2)\left[\frac{s - M_\beta^2}{s} + \frac{M_\alpha^2 + M_\beta^2 - s}{s - M_\alpha^2} \ln\left(\frac{s|M_\alpha^2 + M_\beta^2 - s|}{M_\beta^2 M_\alpha^2}\right)\right]\right\}. \tag{3.3}$$

Here θ is the step function ($\theta(x) = 1$ for $x > 0$ and 0 otherwise), s is the squared center of mass energy, p_1, p_2, p_1' and p_2' are the momenta of t, Q, ℓ_i and N_α respectively, and M_β is the mass of N_β (note that for $\beta = \alpha$ there is no contribution since the product of the relevant Yukawa couplings is real). The overall factor of 6 corresponds to the summation over the gauge degrees of freedom. Note that the term proportional to $\theta(s - M_\beta^2)$ appears due to the possibility of performing a new cut in the one-loop graph in figure $1(b)$ involving the N_β and lepton lines. Since N_β can go on-shell only when the center of mass energy is sufficiently large ($s > M_\beta^2$) this contribution is relevant only for temperatures not much smaller than M_β. Our results hold in the zero temperature limit; in particular we take all the particles, except the heavy Majorana neutrinos, to be massless. At high temperatures, finite temperature effects can induce non negligible corrections to our expressions. In particular, when $M_1 < M_H(T) + M_{\ell_i}(T)$ (implying that decays and inverse decays are blocked [9]) we expect that thermal masses will also have the effect of suppressing the scattering CP violating asymmetries.

Due to crossing symmetry, the CP asymmetry for the processes $N_\alpha \bar{t} \to \bar{Q}\ell_i$ and $N_\alpha Q \to t\ell_i$ in which the Higgs is exchanged in the t-channel can be obtained from the previous result by replacing the Mandelstam variable s by t. Note that for massless quarks $t \leq 0$ so that $\theta(t - M_\beta^2) = 0$ and hence in this case no new cut is present.

The cross sections are obtained by integrating the modulus squared of the invariant amplitudes:

$$\sigma = \frac{1}{64\pi^2(E_1 + E_2)^2} \int \frac{|\mathbf{p_1'}|}{|\mathbf{p_1}|} |\mathcal{M}|^2 \mathrm{d}\Omega_1', \tag{3.4}$$
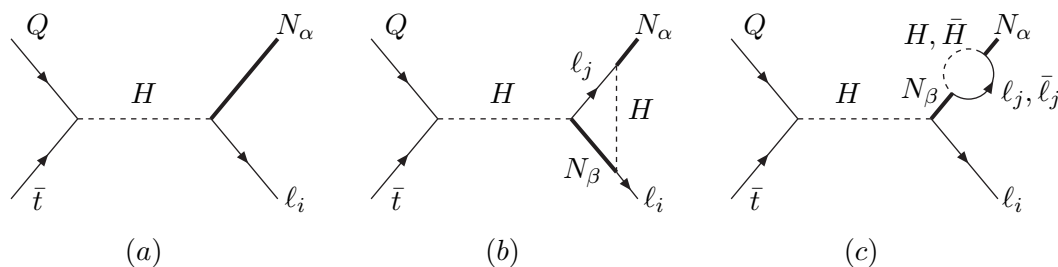
Figure 1: Diagrams contributing to the CP asymmetry in $Q\bar{t} \leftrightarrow N_\alpha \ell_i$ scatterings.

where $p_i = (E_i, \mathbf{p_i})$, $p'_i = (E'_i, \mathbf{p'_i})$ $(i = 1, 2)$ are the momenta of the initial and final particles respectively, and the cross section for the CP conjugate process $\bar{\sigma}$ is defined in the same way. In terms of the cross sections, the CP asymmetry of the s-channel scattering is:

$$
\left[\sigma(Q\bar{t} \to N_\alpha \ell_i) - \bar{\sigma}\right](vertex) = -\frac{1}{8\pi} \frac{\sum_j [\sigma(Q\bar{t} \to N_\alpha \ell_j) + \bar{\sigma}]}{(\lambda^\dagger \lambda)_{\alpha\alpha}} \times
$$

$$
\sum_\beta \mathrm{Im}[\lambda_{i\alpha}\lambda^*_{i\beta}(\lambda^\dagger\lambda)_{\beta\alpha}] \frac{M_\alpha M_\beta}{s - M_\alpha^2} \left\{ \left[1 - \frac{M_\alpha^2 + M_\beta^2 - s}{M_\alpha^2 - s} \ln\left(\frac{|M_\alpha^2 + M_\beta^2 - s|}{M_\beta^2}\right)\right] - \right.
$$

$$
\left. \theta(s - M_\beta^2) \left[\frac{s - M_\beta^2}{s} + \frac{M_\alpha^2 + M_\beta^2 - s}{s - M_\alpha^2} \ln\left(\frac{s|M_\alpha^2 + M_\beta^2 - s|}{M_\beta^2 M_\alpha^2}\right)\right] \right\}. \tag{3.5}
$$

Unlike what happens in the case of decays, the asymmetry $\sigma - \bar{\sigma}$ of the scattering rates now depends on s, so that the convolution necessary to obtain the asymmetry $\Delta\gamma_{N\ell}^{Q\bar{t}}$ of the thermally averaged rates doesn't lead to a simple analytical expression, and has to be performed numerically.

For the CP asymmetry in the t-channel scattering there is no simple analytical expression. Also, the usual infrared divergence appears in the limit $t \to 0$, which can be regularized by replacing the factor $1/t$ coming from the (massless) Higgs propagator by $1/(t - m_H^2)$, and using here the Higgs thermal mass.

The CP asymmetry in scatterings coming from the wave function piece (interference between the diagrams in figures 1(a) and 1(c)) turns out to be always the same as the CP asymmetry for the decays:

$$
\frac{\Delta\gamma_{N_\alpha \ell_i}^{Q\bar{t}}(wave)}{\sum_j (\gamma_{N_\alpha \ell_j}^{Q\bar{t}} + \gamma_{N_\alpha \bar{\ell}_j}^{\bar{Q}t})} = \frac{\Delta\gamma_{\ell_i H}^{N_\alpha}(wave)}{\sum_j (\gamma_{\ell_j H}^{N_\alpha} + \gamma_{\bar{\ell}_j \bar{H}}^{N_\alpha})} \equiv \varepsilon_\alpha^i(wave), \tag{3.6}
$$

where the decay asymmetry is defined in the usual way as $\varepsilon_\alpha^i \equiv \Delta\gamma_{\ell_i H}^{N_\alpha}/\gamma_D^{N_\alpha \to 2}$.

The ratio between the CP violating scattering asymmetries and their approximate expressions in terms of the asymmetry in decays derived in ref. [16], are shown in figure 2 as a function of T/M_1[2]. The results for processes with the Higgs exchanged in the s and

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[2]For definiteness, we neglected in this plot the contribution to the wave part proportional to $M_\alpha(\lambda^\dagger\lambda)_{\alpha\beta}$, keeping the one proportional to $M_\beta(\lambda^\dagger\lambda)_{\beta\alpha}$, in which case vertex and wave contributions become proportional to the same combination of couplings, see ref. [26].
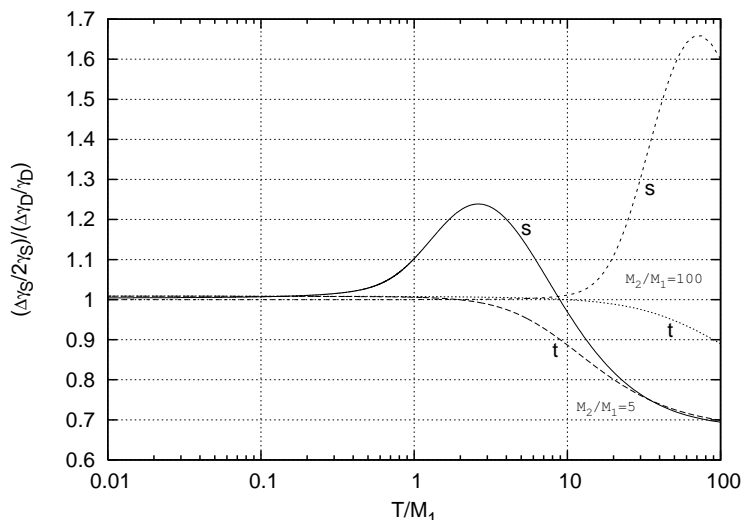
Figure 2: Ratio between the CP violating asymmetry in scatterings $\Delta\gamma_{N\ell_i}^{Q\bar{t}}/2\gamma_{N\ell_i}^{Q\bar{t}}$ and the corresponding quantity for decays $\Delta\gamma_{\ell_iH}^N/2\gamma_{\ell_iH}^N$, vs. T/M_1 for two ratios between the masses of the two lightest heavy neutrinos $M_2/M_1 = 5$ and $M_2/M_1 = 100$. The contributions from s and t-channel Higgs exchange processes are shown separately.

in the t-channels are presented separately. Two illustrative values for the ratio of the two lightest Majorana neutrino masses have been used: $M_2/M_1 = 5$ and $M_2/M_1 = 100$. The effects of N_3 have been ignored (this would correspond either to the case $M_3 \gg M_2$ or to the situation in which the complex phase in the combination of Yukawas associated to N_3 is particularly suppressed). It is apparent that this ratio starts to deviate from unity already for $T > M_2/10$ and that deviations of few tens of percent can appear for T approaching M_2. Since the relevant temperature for leptogenesis in these scenarios is typically $0.1 < T/M_1 < 10$, the approximations adopted in ref. [7, 16] should be good if $M_2/M_1 \gg 10$, while some corrections appear for milder hierarchies at temperatures $T > M_2/10$. It is also easy to show analytically, starting from eq. (3.5), that a factorization for the vertex part, analogous to that of the wave part, is obtained in the limit of M_1 and $T \ll M_2$.

Regarding the scattering processes with gauge bosons, such as $N\ell \to \bar{H}A$, $NH \to \bar{\ell}A$ or $NA \to \ell H$, the associated CP asymmetries can be obtained in a similar way, computing the interferences between tree and one loop scattering amplitudes. One significant difference is that now box diagrams are present, in which the gauge boson is attached to a lepton or Higgs in the loop of the vertex like diagrams, leading to more complicated expressions. The absorptive parts can be obtained using Cutkowski's rule and new cuts appear, but again only for $s > M_\beta^2$, so that in the hierarchical limit the factorized expression holds for $T \ll M_2$, as was the case in the scatterings with quarks discussed before.

The fractional contributions to the source terms in the BE for Y_{Δ_i} arising from decays and scatterings are shown in figure 3 using the factorized expressions in terms of the decay asymmetries. We present separately the results for two body decays $F_D \equiv \Delta\gamma_{\ell_iH}^N/\sum\Delta\gamma$,

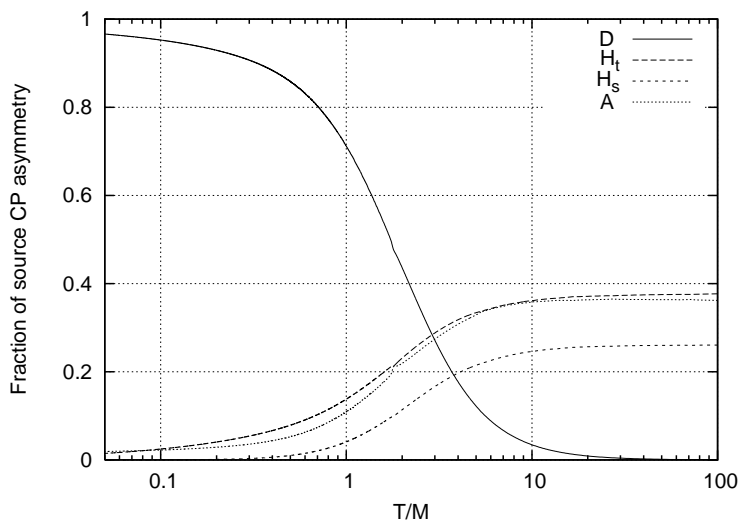**Figure 3:** Fractional contribution to the source terms from the CP asymmetries in decays (D) and in scatterings (s and t channel Higgs exchange and scatterings involving gauge bosons) for $M_2 \gg M_1$, obtained in the zero temperature approximation.

s-channel $F_{H_s} \equiv \Delta\gamma_{\ell_i N}^{Q\bar{t}} / \sum \Delta\gamma$, t-channel Higgs exchange processes $F_{H_t} \equiv 2\Delta\gamma_{\ell_i t}^{NQ} / \sum \Delta\gamma$ and the gauge boson contribution[3] $F_A \equiv (\Delta\gamma_{\ell_i H}^{NA} + \Delta\gamma_{\ell_i A}^{N\bar{H}} + \Delta\gamma_{AH}^{N\bar{\ell}_i}) / \sum \Delta\gamma$ where

$$\sum \Delta\gamma \equiv \Delta\gamma_{\ell_i H}^{N} + \Delta\gamma_{\ell_i N}^{Q\bar{t}} + 2\Delta\gamma_{\ell_i t}^{NQ} + \Delta\gamma_{\ell_i H}^{NA} + \Delta\gamma_{\ell_i A}^{N\bar{H}} + \Delta\gamma_{AH}^{N\bar{\ell}_i}, \tag{3.7}$$

while the contributions from decays into three body final states are always negligible and are not shown (see the appendix). These fractions are independent of the value of the neutrino Yukawa couplings adopted and, in the hierarchical limit in which the factorization is valid, they are also independent of the value of M_2/M_1. From figure 3 it is seen that scattering CP asymmetries are the dominant source term for $T > 2M_1$, and hence can play an important role in the early leptogenesis phase.[4]

4. Results

In figure 4 we display the results of the integration of the BE adopting $M_1 = 10^{11}$ GeV and $\tilde{m}_1 \equiv v^2(\lambda^\dagger\lambda)_{11}/M_1 = 0.06$ eV (where v is the Higgs VEV). To avoid complications with τ-flavor effects that are active in this temperature regime, we also assume a flavor 'aligned' situation in which the lepton doublet ℓ_1 to which N_1 decays has no τ flavor component, that is $K_\tau \equiv |\langle\ell_1|\ell_\tau\rangle|^2 = 0$ (and similarly for $\bar{\ell}_1$). The dashed line corresponds to the case

---

[3]The contribution of gauge boson scatterings has been estimated using the expressions given in [9].

[4]When thermal masses are taken into account, at very high temperatures ($T \gtrsim 7M_1$) the condition $M_H(T) > M_1 + M_{\ell_i}(T)$ is met and the decay $H \to N_1\ell_i$ can occur. Since the asymmetry for this decay has a large enhancement from thermal effects [9], in this temperature regime actually the Higgs decays would become the dominant source of the lepton asymmetry.
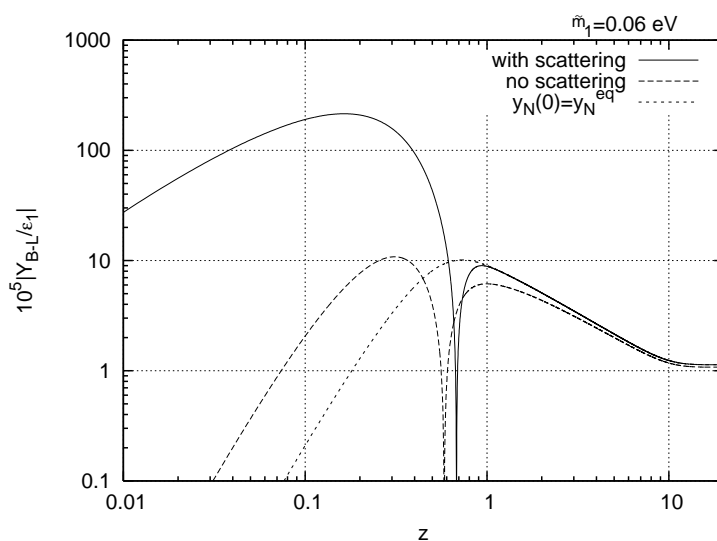
Figure 4: Evolution of the $B - L$ asymmetry as a function of $z = M_1/T$ with the CP asymmetries in scatterings neglected (dashed line) and included (solid lines). The scattering asymmetries are approximated by using the asymmetry in decays. This figure corresponds to a strong washout regime with $\tilde{m}_1 = 6 \times 10^{-2}\,\mathrm{eV}$. For comparison, the value that would be obtained starting with an equilibrium N density is also displayed (dotted line).

in which the contributions of the CP scattering asymmetries to the source term are ignored (see ref. [11] for more details), while the solid line depict the results obtained by including the CP asymmetries of the scattering processes, adopting their factorized expression (using the exact expressions would slightly modify the asymmetries for $T > M_1$, but the final values would be almost unchanged). The dotted line is the asymmetry that would result had the initial density of N be the equilibrium one. It is apparent that for $T > M_1$ the scattering processes have a large effect in the production of the lepton asymmetry. This example corresponds to a case of strong washout, with $\tilde{m}_1 \gg 10^{-3}\,\mathrm{eV}$. In particular, we see that in this case the washouts affect the evolution of the lepton asymmetries up to $z \simeq 10$. Hence, even if at early times large additional sources of CP violation are present, late washouts turn out to be decisive in determining the final asymmetry, which ends up being equal to the one that would be obtained had one started with $Y_N = Y_N^{\mathrm{eq}}$. This also means that in this regime the final asymmetry becomes essentially independent from the conditions at early times.

We recall here that a crucial point in thermal leptogenesis is that if the washout processes were switched off completely during the whole leptogenesis phase (and if the dependence of the CP asymmetries on the temperature that is induced by thermal effects was also ignored) the inclusion in the BE of the sources of CP violation from scatterings would yield a zero final asymmetry [16]. This can be seen by writing the different $\Delta L_i = 1$

scattering CP asymmetries in terms of their approximate expressions, i.e.

$$\Delta\gamma_Y^{NX} \simeq \frac{\gamma_Y^{NX}}{\gamma_{\ell_i H}^N}\Delta\gamma_{\ell_i H}^N. \tag{4.1}$$

Then, the source term in the BE for Y_{Δ_i} can be rewritten just in terms of the decay asymmetry $\Delta\gamma_{\ell_i H}^N$ as

$$(\dot{Y}_{\Delta_i})_s \simeq -(y_N - 1)\frac{\Delta\gamma_{\ell_i H}^N}{\gamma_{\ell_i H}^N}\sum_{\Delta L_i = 1}\gamma_Y^{NX}. \tag{4.2}$$

By using the relation

$$\frac{\gamma_{\ell_i H}^N}{\sum_j(\gamma_{\ell_j H}^N + \gamma_{\bar{\ell}_j \bar{H}}^N)} = \frac{\sum_{\Delta L_i = 1}\gamma_Y^{NX}}{\sum_{X,Y}\gamma_Y^{NX}}, \tag{4.3}$$

where the sum in the denominator in the r.h.s. is over all the processes γ_Y^{NX} for which $|\Delta L_j| = 1$ and over all the flavors j, and defining the flavor asymmetry $\varepsilon_1^i = \Delta\gamma_{\ell_i H}^N/\sum_j(\gamma_{\ell_j H}^N + \gamma_{\bar{\ell}_j \bar{H}}^N)$ we can finally rewrite the source term as

$$(\dot{Y}_{\Delta_i})_s \simeq -(y_N - 1)\,\varepsilon_1^i\sum_{X,Y}\gamma_Y^{NX}. \tag{4.4}$$

Combining now the above expression with the BE for Y_N we obtain

$$\dot{Y}_N - \frac{\dot{Y}_{\Delta_i}}{\varepsilon_1^i} \simeq -\frac{(\dot{Y}_{\Delta_i})^w}{\varepsilon_1^i}. \tag{4.5}$$

In the absence of washouts the r.h.s. of the equation above would then vanish, and in the approximation in which ε_1^i is taken as independent of the temperature, the quantity $Y_N - Y_{\Delta_i}/\varepsilon_1^i$ would hence be constant. Then, for thermal leptogenesis scenarios in which the N density and the lepton asymmetries vanish initially, this quantity will just be zero, showing that the Y_{Δ_i} asymmetries generated at early times are erased at later times as the N_1 disappear by decays or scatterings. Then, as was pointed out in [9], any effect that breaks this cancellation, as for example a dependence of the CP asymmetries on the temperature, could be numerically important. The cancellation will no longer hold also if some washout processes are particularly efficient at temperatures $T > T_0$, where T_0 is the temperature at which the lepton asymmetry changes sign (note that this type of washouts could yield an enhancement in the final asymmetry, while in general late washouts at $T < T_0$ always tend to reduce its final value). Of course, in the cases when the heavy N states are produced through other processes not related to the ones giving rise to the CP asymmetries, such as via scatterings involving heavy right-handed W or additional Z' bosons, or are produced non-thermally via e.g. inflaton decays, the leptogenesis initial condition $Y_N(0) \neq 0$ would directly prevent the cancellation of the lepton asymmetry.

In other words, the origin of the cancellation can be understood as follows. At any time there are three kinds of possible sources for the lepton asymmetries: off-shell scatterings, processes producing real Ns and processes in which Ns are destroyed. In general, the lepton asymmetry produced by the off-shell scatterings is twice as large, and with opposite
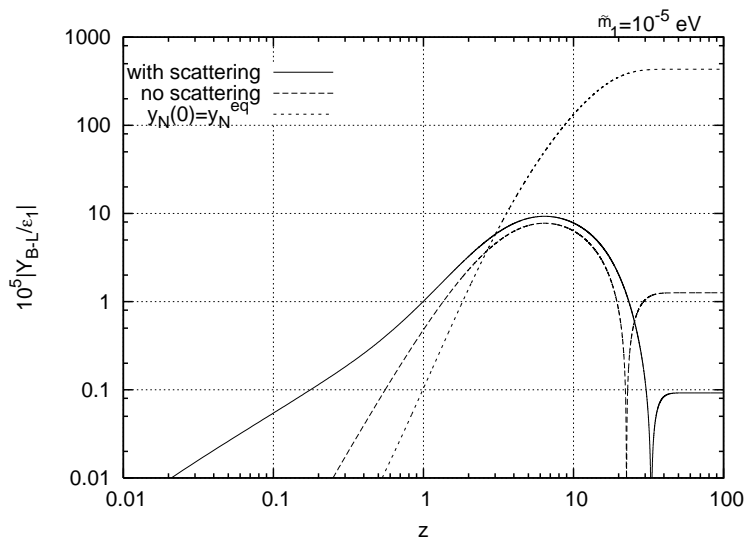
Figure 5: Same as figure 4 but in a regime of weak washout, with $\tilde{m}_1 = 10^{-5}\,\mathrm{eV}$.

sign, as that associated to processes in which real Ns are produced, but that associated to processes in which real Ns are destroyed depends on the N density Y_N. If Y_N equals the equilibrium density Y_N^{eq}, the asymmetry of N production and destruction processes are equal, and hence the sum of the three asymmetries cancel. On the other hand, if $Y_N < Y_N^{\mathrm{eq}}$ the asymmetry produced by off-shell processes dominates, while the opposite happens if $Y_N > Y_N^{\mathrm{eq}}$. This is why the sources of lepton asymmetry in the BE are just proportional to $Y_N - Y_N^{\mathrm{eq}}$. Now, when the factorization of the scattering asymmetries in terms of the decay asymmetry holds, one finds that the sources of the lepton asymmetries are just ϵ_1 times the sources of the N density (assuming that no other processes besides the Yukawa couplings produce Ns). This means that if ϵ_1 is constant (T independent) and we ignore the washout processes, the total integrated change in the lepton asymmetry will be just ϵ_1 times the total change in Y_N. Hence, if this last vanishes, as is the case when the initial condition is that of vanishing N density, the final leptonic density would also vanish.

Clearly, the impact of including CP scattering asymmetries in the BE is qualitatively different in the strong and in the weak washout regimes. In the weak washout regimes (corresponding to values of $\tilde{m}_1 < 10^{-3}\,\mathrm{eV}$) the effect of late washouts is negligible, the asymmetry is strongly affected by the cancellation and thus its final value turns out to be rather sensitive to the amount of washouts in the early phases. This is illustrated in the example in figure 5, that corresponds to the value $\tilde{m}_1 = 10^{-5}\,\mathrm{eV}$. In this figure we compare the evolution of the asymmetries in the two cases when the CP asymmetries of the processes involving the top-Yukawa and the gauge interactions are included or are left out (all scatterings are in any case included as sources for N production). It is apparent that when the sources of CP violation from scatterings are included the effects of the cancellation strongly reduce the final asymmetry obtained. Since only weak washouts are present, the cancellation remains quite effective and the final value of the asymmetry is rather small. In
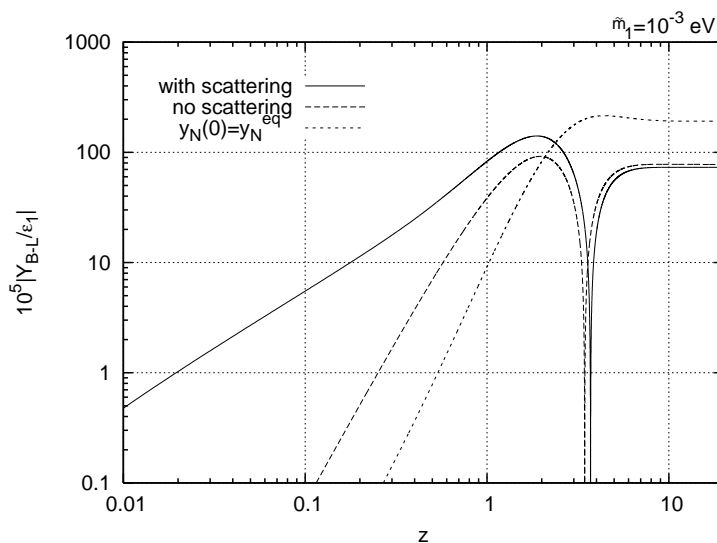
Figure 6: Same as figure 4 but in a regime of intermediate washout, with $\tilde{m}_1 = 10^{-3}\,\mathrm{eV}$.

this regime, the final asymmetries are also much smaller than the asymmetries that would result starting with an equilibrium density for the N's since in that case no asymmetry is generated at early times, and no cancellation can occur.

Finally, figure 6 gives an example with an intermediate washout strength $\tilde{m}_1 = 10^{-3}\,\mathrm{eV}$. In this case an intermediate behavior between those of the weak and strong washouts is observed: similarly to the strong washout case, the final asymmetry remains almost unchanged whether the CP scattering asymmetries are included or not. However, as in the case of weak washouts, its value remains well below what would be obtained by starting with an equilibrium density of N's.

5. Conclusions

In this paper we have computed the CP asymmetries of scattering processes involving the top quark and the gauge bosons. CP violation in scatterings gives an important contribution to the generation of a lepton asymmetry at high temperatures ($T \gtrsim 2M_1$), and in particular in the zero temperature approximation adopted in our calculations this contribution is by far the dominant one. We have compared our results with the approximate expressions of the scattering CP asymmetries in terms of the decay asymmetry, concluding that in scenarios in which the heavy Majorana neutrino masses are sufficiently hierarchical this approximation provides reasonably accurate results. We have shown that when the sources of CP violation in scatterings are included in the BE, in the limit of very weak washouts a strong cancellation between the asymmetries generated at early times (when $Y_N < Y_N^{\mathrm{eq}}$), and the asymmetry of opposite sign generated at later times (when $Y_N > Y_N^{\mathrm{eq}}$) takes place, sizably suppressing the final lepton asymmetry with respect to the cases in which CP asymmetries in scatterings are neglected.

In the strong washout regimes, for which the final lepton asymmetry is almost independent of the conditions at early times, such as the initial value of the right-handed neutrino density, the final results are instead essentially unaffected by the inclusion of the new sources of CP violation.

Acknowledgments

A. Three body N decay

The three body decay width for $N_1 \to \ell_i \overline{Q}_3 t$ is

$$
\Gamma(N_1 \to \ell_i \overline{Q}_3 t) = \frac{3}{16\pi^2} h_t^2 \Gamma_0(N_1 \to \ell_i H) \int_{(\sqrt{a_t} + \sqrt{a_Q})^2}^{(1 - \sqrt{a_\ell})^2} \mathrm{d}x \, \frac{(x - a_t - a_Q)(1 + a_\ell - x)}{(x - a_H)^2 + a_H c_H} \times
$$
$$
\times \frac{1}{x} \left(\left[(x - a_Q + a_t)^2 - 4x a_t \right] \left[(1 - x - a_\ell)^2 - 4x a_\ell \right] \right)^{1/2}, \qquad (A.1)
$$

where $a_y \equiv (m_y/M_1)^2$ and $c_H \equiv (\Gamma_H/M_1)^2$, with Γ_H being the decay width of the Higgs boson. $\Gamma_0(N_1 \to \ell_i H) = |\lambda_{i1}|^2 M_1/16\pi$ is the two body decay width of N_1 (at zero temperature) and the integration variable is $x = (p_t + p_Q)^2/M_1^2$, where p_t and p_Q are the momenta of t and $\overline{Q}_3$ respectively.

We assume $m_t + m_{Q_3} > m_H$, which is generally valid at high T if thermal masses are considered and also at $T = 0$ if the Higgs boson is not too heavy. In this case the Higgs boson exchanged in the internal line of the three body decay cannot be on-shell, and the Higgs width (parametrised by c_H) can be neglected. (Note that a resonant contribution would in any case correspond to the two body decay $N_1 \to \ell_i H$ rather than to a genuine three body process.) In the zero temperature limit a_t, a_Q, $a_H \to 0$ the integral in eq. (A.1) would get a large enhancement from the region corresponding to small values of x. However, for finite values of the thermal masses this enhancement is not present, and in particular for $T/M_1 > 10^{-2}$ the three body decay rate is always less than 6% of the two body decay rate. Note also that, due to the effects of thermal masses, the phase space for both decays actually gets closed when T approaches M_1 [9].

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